MIDDLE POMERANIAN SCIENTIFIC SOCIETY OF THE ENVIRONMENT PROTECTION
ŚRODKOWO-POMORSKIE TOWARZYSTWO NAUKOWE OCHRONY ŚRODOWISKA

Annual Set The Environment Protection
Rocznik Ochrona Środowiska

Volume/Tom 15. Year/Rok 2013 ISSN 1506-218X 405–419

Analytical Model for Vertical Velocity Distribution and Hydraulic Roughness at the Flow Through River Bed and Valley with Vegetation

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1. Introduction

While analyzing river and valley environment, it should be remembered that the river itself is only a component of a larger ecosystem of a river with its valley. Biogenetic relationships between river and floodplain are anlysed as a whole ecosystem of a river valley. It is of a considerable importance for the function of a complex biocenosis. Both components of the system a so closely related that while studying functions changes and dangers of one, the other has to be taken into account.

On the banks of the main stream channel and the floodplain there is a great variety of vegetation: grass, perennials, reeds, bushes and trees. Taking into consideration the depth of the water, vegetation may be divided into submerged and emergent. Emergent vegetation includes most often trees and large bushes of the height of 3.0 to 5.0 m. Where water is not deep on the floodplains, also high perennials and reeds may be emergent. As the level of water changes naturally, in practice the same vegetation may be defined in different ways. Vertical velocity distribution in the area with submerged vegetation may be described with a logarithmic velocity distribution, because, as measurements in channels show, in this area velocity does not change with depth. In the case of submerged vegetation, a characteristic feature of vertical velocity distribution is the presence of a zone shaped by this vegetation and logarithmic distribution above the vegetation.

The presence of various types of vegetation in the same place leads to creation of two or three zones of various vegetation densities. The highest density of the vegetation structure is observable in the bottom zone (the height of about 0.4 to 0.5 m), where there are grasses, reeds and bushes at the same time. A little lower density of vegetation is observable in the middle zone of the height of 0.5 to 1.0 m where there are reeds and bushes. The lowest is the density of the upper zone, above the tops of reeds. Therefore, apart from the degree of submergence and deformation caused by the flow, the density of the vegetation structure is a significant element shaping the velocity distribution. Characteristic parameters of vegetation are calculated on the basis of direct measurements. The distribution of plants on floodplain is defined on the basis of aerial and satellite photographs or during stocktaking in situ.

In the paper, a new model of vertical velocity distribution and hydraulic roughness is presented, taking into consideration various submerged and emergent vegetation. This model was based on the balance of forces present during the water flow in the area with vegetation cover.

2. Materials and methods

Water flow above the area covered with reeds and bushes was analysed. In the lower zone of bushes of the height of about 1.0 m. there are reeds that during the spate are not subject to deformation. In this case there appear two levels of various density of the vegetation structure (fig.1). The velocity distribution in the area with vegetation is calculated on the basis of the balance of forces, described with the equation (1). At the depth of flow h there are three layers: 1 (bottom), 2 (lower) and 3 (upper). There are transitional layers between them. The velocity profile is treated separately for layer 2 (lower), where the density of elements is higher and level 3 (upper), of lower density of vegetation elements. In layer 1 (bottom) the vertical momentum exchange is taken into account. It is caused by the friction of water at the bottom. Profiles in the layers 1, 2 and 3 are adjusted to the surface of division by adequate boundary conditions. In layer 1 (bottom), in the scope of changeable ($ks<z<lm$), velocity distribution is defined with the equation (1):

$$u_1(z) = \left[\frac{1}{2} C_2 e^{z\sqrt{2A_1}} - \frac{M}{C_2 e^{z\sqrt{2A_1}}}\right](l_s - z) \tag{1}$$

where:

$$M = \frac{C_1}{(1+\kappa^2)A_{1m}} - (\bar{u}_2)^2 \tag{2}$$

$$A_1 = \frac{C_w m_2 D_2}{2\kappa^2} \tag{3}$$

$$\bar{u}_2 = \sqrt{\frac{2gI}{C_w m_2 D_2}} \tag{4}$$

In the transitional zone, in the lover part of layer 2 (lower) (fig. 1), as far as changeability is concerned $[l_m<z<(l_m+z_{01})]$, velocity u_2 is expressed by following relationship:

$$u_2 = \sqrt{\bar{u}_2^2 - \left[C_3\, e^{z_1\sqrt{2A_2}} + C_4\, e^{-z_1\sqrt{2A_2}}\right]} \tag{5}$$

where:

$$A_2 = \frac{C_w m_2 D_2}{2\alpha_2} \tag{6}$$

In layer 2 (bott) upper the bottom zone 2g is present and the momentum conservation equation has the following form:

$$u_{2g}(z_2)\frac{d^2 u_{2g}(z_2)}{dz_2^2} + \left(\frac{d u_{2g}(z_2)}{dz_2}\right)^2 = \frac{C_w m_2 D_2 \left[u_{2g}(z_2)\right]^2}{2\alpha_2} - \frac{gI}{\alpha_2} \tag{7}$$

where: $u_{2g}(z_2)$ – velocity in the distance z from the bottom, in upper transitional zone, in zone 2 (bottom), z_2 – the replacement variable is defined with the equation:

$$z_2 = z - (k - z_{02}) \tag{8}$$

where: k – the height of the bottom layer of larger density of the vegetation elements (bushes and reed), z_{02} – the height the transitional zone in the upper parts of layer 2 (bottom). The equation (7) has the following analytical solution:

$$u_{2g}=\sqrt{\frac{2}{E_1}}\left[C_5 e^{z_2\sqrt{2A_2}}-C_6 e^{-z_2\sqrt{2A_2}}\right] \tag{9}$$

where: A_2 – constant is presented in the following equation (6), z_2 – variable, defined with the equation (8), u_{2g} – is velocity in distance z from the bottom, $[(k–z_{02}) \le z \le k]$.

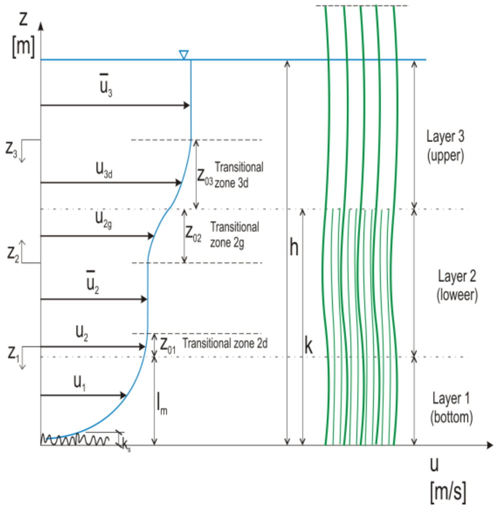

Fig. 1. Vertical velocity distribution in the flow with emergent vegetation of vegetation elements with various densities.
Rys. 1. Pionowy rozkład prędkości z roślinnością nie zatopioną o różnej gęstości elementów roślinnych.

The height of the transitional zone z_{02} is the function of the density of the vegetation structure (fig.1) in layer 2 (bottom) and layer 3 (upper) and is expressed by the following relationship:

$$z_{02}=2a_{x2}\left(1-\frac{m_3 D_3}{m_2 D_2}\right) \tag{10}$$

where: m_2 and m_3 – density of the vegetation structure, in layer 2 (bottom) and layer 3 (upper), D_2, D_3 – diameter of the vegetation elements, in the direction of the flow in layers 2 and 3, a_{x2} – distance between the vegetation elements, in the direction of the flow in layer 2 (bottom). Constant of integration C_5 and C_6 result from the boundary conditions. The first boundary condition is at the height of $z = k - z_{02}$, that is $z_2 = 0$ and takes the following form:

$$u_{2g}\Big|_{z_2=0} = \bar{u}_2 \tag{11}$$

and from equation (9) we obtain:

$$C_5 = -C_6 \tag{12}$$

At the top of the vegetation layer of lager density of the elements, that is the surface of the distribution area, between layers 2 (bottom) and 3 (upper), the present velocity must be equal and that is why the second boundary condition takes the following form:

$$u_{2g}\Big|_{z_2=z_{02}} = u_{3g}\Big|_{z_3=z_{03}} \tag{13}$$

where: z_2 – the replacement variable, defined with the equation (8), z_{02} – the height of the transitional zone in the upper parts of layer 2 (bottom), – z_3 – replacement variable, and may be presented in the form of the following equation:

$$z_3 = (k + z_{03}) - z \tag{14}$$

where: z_{03} – the height of the transitional zone in layer 3 (upper) and may be presented in the form of the following equation:

$$z_{03} = 2a_{x3}\left(1 - \frac{m_3 D_3}{m_2 D_2}\right) \tag{15}$$

where a_{x3} is the distance between the vegetation elements in the direction of the flow in layer 3 (upper). Density of the structure of the vegetation elements in layer 3 (upper) m_3 is expressed by the following relationship:

$$m_3 = \frac{1}{a_{x3} a_{y3}} \tag{16}$$

with the assumption, that $a_{x3} = a_{y3}$, we obtain:

$$a_{x3} = \sqrt{\frac{1}{m_3}} \tag{17}$$

where a_{y3} is the distance between the vegetation elements in cross direction to the flow in layer 3 (upper).

After substitution in the place of the left side of the boundary condition (13) the equation (9), the constant C_5 takes the form of:

$$C_5 = \frac{\left(u_{3k}^2 - u_2^2\right)}{E_4} \tag{18}$$

where:

$$E_4 = e^{z_{02}\sqrt{2A_2}} - e^{-z_{02}\sqrt{2A_2}} \tag{19}$$

Velocity u_{3k} in the boundary condition (13) and equation (18) is obtained from the equation of velocity distribution in the level 3 (upper), that is presented below.

In the layer 3 (upper) the bottom transitional zone 3d is present (fig. 1) and the turbulent shear stress is described equation may be presented as follows:

$$u_{3d}(z_3)\frac{d^2 u_{3d}(z_3)}{dz_3^2} + \left(\frac{d u_{3d}(z_3)}{dz_3}\right)^2 = \frac{gJ}{\alpha_3} + \frac{\left[\ \right]}{2\alpha_3} \tag{20}$$

where: $u_{3d}(z_3)$ – velocity in distance z from the bottom in the transitional zone in layer 3 (upper), z_3 – replacement variable, z_{03} – height of the (bottom) transitional zone, in layer 3 (upper) of lower density of the vegetation, $\alpha_3 = \kappa \cdot a_{x3}$ – characteristic scale of the length of macro-turbulence in layer 3 (upper). The equation (20) has the following analytical solution:

$$u_{3d} = \sqrt{-\frac{2}{t_3} + \left[C_7 e^{z_3\sqrt{2A_3}} + C_8 e^{-z_3\sqrt{2A_3}}\right]} \tag{21}$$

where: A_3 – constant presented in the following equation:

$$A_3 = \frac{C_w m_3 D_3}{2\alpha_3} \tag{22}$$

$$\bar{u}_3 = \sqrt{\frac{2gI}{C_w m_3 D_3}} \tag{23}$$

and: u_{3d} – velocity in the distance z from the bottom [$k \le z \le (k + z_{03})$], $\bar{u}_3$ – characteristic velocity constant in layer 3-upper with cover of vegetation elements. It results from the equation (20). Constants C_7 and C_8 result from boundary conditions. The first boundary conditions the transitional zone in layer 3 (upper), at the height of $z = k + z_{03}$ that is $z_3 = 0$ and has the following form:

$$u_{3d}\Big|_{z_3=0} = \bar{u}_3 \tag{24}$$

And from equation (21) we obtain:

$$C_7 = -C_8 \tag{25}$$

In the distribution layer 2 and 3, with different plant density the present velocity gradients must be equal. At the height $z = k$, that is $z_3 = z_{03}$, the second boundary condition takes the form of:

$$\frac{\partial u_{2g}}{\partial z_2}\Big|_{z_2=z_{02}} = \frac{\partial u_{3d}}{\partial z_3}\Big|_{z_3=z_{03}} \tag{26}$$

Constant C_7 takes the form of:

$$C_7 = \frac{l^2 E_6^2}{A_3 X_3^2 E}\left[\sqrt{1 + \frac{2 A_3 X_3^2 \bar{u}_3^2}{l^2 E_6^2}} - 1\right] \tag{(27)}$$

where:

$$T = \frac{\sqrt{2A_2}\,E_4^{'}\,C_5}{2\sqrt{\bar{u}_2^2 + E_4 C_5}} \tag{28}$$

$$E_4^{'} = e^{z_{02}\sqrt{2A_2}} + e^{-z_{02}\sqrt{2A_2}} \tag{29}$$

$$E_5 = e^{z_{03}\sqrt{2A_3}} + e^{-z_{03}\sqrt{2A_3}} \tag{30}$$

$$E_6 = e^{z_{03}\sqrt{2A_3}} - e^{-z_{03}\sqrt{2A_3}} \tag{31}$$

and: A_2, A_3, $\bar{u}_2$, $\bar{u}_3$, z_{02}, z_{03}, – values take the form of (6), (22), (4), (23), (10), (15).

The velocity on the surface of the division of two layers: 3 and 3, u_{3k}, present in the boundary condition (26) and equation (18), is obtained from the equation of velocity distribution in layer 3, and presented in the form of equation (21). When the statement (25) is used:

$$u_{3k} = \sqrt{\bar{u}_3^2 - C_7 E_6} \tag{32}$$

Constant C_7 takes the form of:

$$C_7 = \frac{K - E_4 C_5}{E_6} \tag{33}$$

where:

$$K = \bar{u}_3^2 - \bar{u}_2^2 \tag{34}$$

The constant of integration C_5 is calculate from the 4th degree equation with consecutive approximations:

$$aC_5^4 + bC_5^3 + cC_5^2 + dC_5 + e = 0 \tag{35}$$

The mean velocity in the hydrometric vertical equals total particular areas. The velocity with the flow through emergent vegetation with different densities of vegetation elements in two layers (fig.1), may be presented in the following way:

$$U = \frac{1}{h}\left\{\begin{array}{l} \frac{C_2}{4}\left[kE_5 + \left(l_m - k_s\right)\left(E_6 + E_1\right)\right]\left[k l_m - \frac{1}{2}\left(z_{01} + z_{02}\right)\right] u_2 + \\ \frac{z_{01}}{2}\sqrt{u_2^2 - C E_3 E_2} + \frac{z_{02}}{2}\sqrt{u_2^2 + C E_5 E_4} + \\ \frac{z_{03}}{2}\sqrt{u_3^2 - C E_7 E_6}\left(hk - \frac{1}{2} z_{03}\right) u_3 \end{array}\right\} \quad (36)$$

For known value of mean velocity in a vertical flow, the value of velocity coefficient may be calculated from the Chezy`s formula or roughens coefficient from the Maninng's equation.

3. Conclusions

3.1. The measurements in concrete, hydraulic channel with stiff emergent elements

The exactness assessment of the proposed analytical model was made on the basis of comparison with the results of the measurements of vertical flow velocity:

1) The floodplains of the Warta river at km 505+570 on 5 April 2006, with the flow of Q =185 m^3/s. The detailed description of study and results was published in [4] and [5].
2) In the concrete, straight open channel with trapezoidal cross-section of the length of 16 m and total width 2.1. The detailed description of study and results was published in [2] and [3].

In all the respective instances in the calculation the constant value B = 2.0 and von Karman's constant κ = 0.4 m were adopted as well as the resistance coefficient in the post-vegetation period C_w = 1.0 m.

3.1.1. The measurements in floodplains of the Warta river with emergent vegetation

The measurements of velocity distribution were made in 5 hydrometric verticals placed inside a large gathering of bushes, reed and sedge. The gathering of vegetation occupies the area between the left embankment of the river and the river bed in the distance of 3.0 km above the Jeziorsko reservoir. On the day of measurements (5 April

2006) with the spring spate flow of the volume of Q = 185 m^3/s, the depth of water in the hydraulic verticals was 1.75 m. The height of the bushes was from 3.0 m to 5.0m and the height of reed from 0.85 m to 1.0m. The velocity measurements were conducted with the use of hydraulic current meter with the diameter of 100 mm. During the velocity measurements the measurement of water level down the left embankment was carried out in section from 503+660 km to 511+400 km. The water level longitudinal slope was measured at the total length of the gathering of vegetation adjacent to the slope drain and equaled I = 1.364 ‰. After the spring spate on 20 April 2006 the measurements of the vegetation structure were made directly above the hydrometric verticals in which the velocity measurements were made on 5 April 2006. The diameters' vegetation and vegetation elements density were measured of three levels. Additional the rough surfaces at the bottom were estimated.

The density of vegetation elements in 2-bottom layer equaled m_2 = 79.1 piece/m^2, and in the layer 3-upper equaled m_3 = 36.4 piece/m^2. The density of the vegetation elements in layer 2-bottom was D_2 = 0.005 m., a w layer 3-upper was D_3 = 0.0094 m.

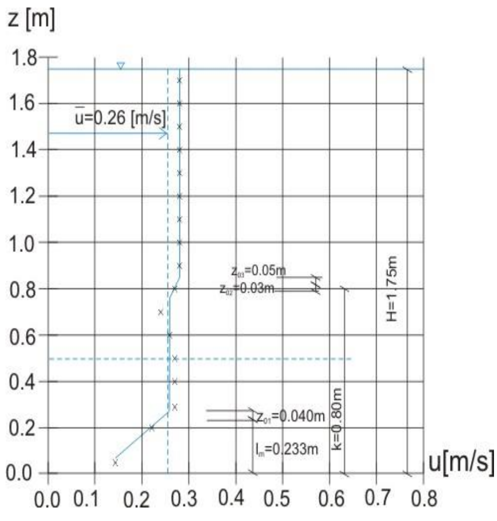

Fig. 2. Measured and calculated flow velocity profile of the Warta River flood planes at km 505+570. vertical no IV

Rys. 2. Pomierzony I obliczony rozkład prędkości na terenach zalewowych rzeki Warty w km 505+570 pion nr 4.

The scale of the length of macro turbulences triggered off be eddies moving longitudinally which may be named "longitudinal mixing length" equaled in layer 2-bottom, α_2 = 0.04 m, and in layer 3-upper α_3 = 0.07m. The obtained results from measurements in situ used from verification results with analytical model and get good compatibility calculations and measurements values velocity (fourth hydraulic vertical fig. 2).

3.1.2 The measurements in concrete, hydraulic channel with stiff emergent elements

In the floodplain of hydraulic channel stiff, metal pipes of 8.0 mm diameter were set. Stiff elements were placed in the knots of a square mesh. The velocity measurements were made on the flow through emergent element. The measurements were made in the Department of Hydraulic Engineering and environmental restoration at Warsaw University of Life Sciences. The measurements' capacity in the rectilinear, concrete hydraulic channel were carried out in two versions:

1) Smooth surface main channel of average roughness of the floodplain k_s = 0.0005 m. Longitudinal slope of the channel bed and water surface equaled I = 0.6‰.
2) Smooth surface main channel and rough surface of the floodplain with average roughness of the floodplain k_s = 0.003m. Longitudinal slope of channel bed and water surface equaled I = 0.4‰.

The depth of water equaled h = 0.134 m. The density of vegetation elements equaled m_2 = 92 m^2/pieces and their diameter D_2 = 0.008 m. The scale of macro-turbulence length induced by horizontal eddies equaled α_2 = 0.04 m. The results received from the distribution velocity measurements were compared with the results obtained with the analytical model. The results obtained from measurements and calculated velocity distribution velocity for the main channel and the channel with greater roughness were in agreement with each other (fig. 3).

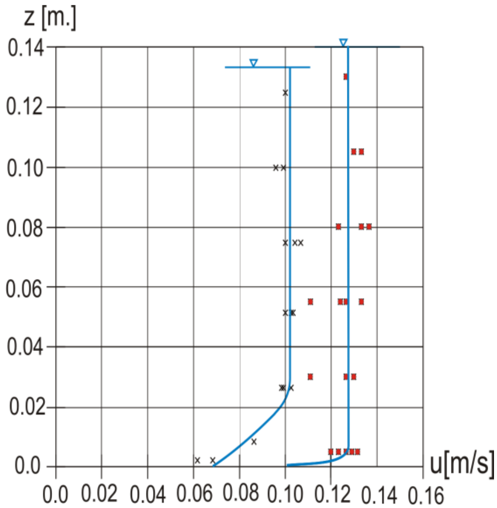

Fig. 3. Measured and calculated flow velocity profile. Concrete, rectilinear flume with smooth bottom [2]

Rys. 3. Pomierzone I obliczone rozkłady prędkości w prostoliniowym, betonowym korycie hydraulicznym o chropowatej powierzchni dna [2]

The intensive development of various vegetation in the natural conditions e.g. grass, perennials, sedge bushes and trees, that is present along the riverbanks, oxbow lakes and due to lack of agricultural exploitation of the floodplains, in the river valley. The dense cover of vegetation elements influences considerably the increase of resistance of the flow. It is caused by vegetation elements of various heights and degree of submergence and various degree of flexibility. In natural conditions of floodplains, different types of plants (grass, sedge, reed, bushes and tress) are grouped together along the river banks and floodplains. These are plants of various height and degree of submergence and different deformation grade. The presence of different types of vegetation in this place leads to creation of two or three levels of various vegetation structures and flow depth so apart from the degree of submergence and deformations caused by the flow one of the essential issues is the vegetation structure density at different depths of the flow.

Solutions presented in literature describing velocity distribution. It was verified through a comparison with results in field measurement in hydraulic channels. The research in hydraulic channels was made at small depths with artificial elements, of the built -in the smooth channel bed. Therefore, in suggested solutions the influence of roughness on velocity distribution was ignored. Observations and measurements in situ in the natural channel of the Warta river displayed multi-layer vegetation structure with the flow above the bottom roughness. The height of the bushes in the floodplain of the Warta river significantly exceeds the depth of the water even during large flood discharge. The observations in situ showed that branches of the bushes do not undergo deformations and may be treated as stiff elements.

On the basis of conducted research and comparative analyses of measured and calculated values of velocity the following conclusions may be drawn:

1) The measurements of velocity distribution in the natural vegetation gathering of the floodplain of the Warta river proved that at the depth of the flow the velocity values are constant and only in the bottom layer there occur significant values of velocity gradients. This velocity distribution at the depth occurs with the flow through emergent homogenous vegetation
2) In the natural vegetation gathering with homogenous vegetation in each layer velocity has constant values. The value of characteristic constant velocity at the bottom layer of greater density of vegetation structure, is lower than the value of this velocity in the upper layer, in which the vegetation density structure is smaller. Between these layers there occur transitional zones, in where the values of velocity gradients are larger than zero.
3) The height of transitional zones between layers depends on density and diameter of the vegetation elements in both layers as well as on the distance between vegetation elements in longitudinal and vertical direction.
4) The distribution of velocity in the bottom layer depends on vertical and horizontal momentum exchange caused by friction at the bottom and flow around vegetation elements. Hence the height of the bottom layer is the function of the height roughness, density and diameter of vegetation elements and coefficient of roughness caused by the flow around the vegetation elements.
5) Transitional zone occurs directly above the bottom layer. The height of this zone is the function of height of the bottom zone and constant value of von Karman.

6) In the calculations of velocity distribution with the flow through stiff elements vegetation without leaves the value of the resistance coefficient $C_W = 1.0$ may be adopted [1].
7) In the analytical solution it was accept that horizontal scale of the length of macro-turbulences depends on the density of the structure of vegetation elements. The scale of length of macro-turbulence may be construed as "horizontal mixing length". This value is the function of the distance between vegetation elements and Karman's constant.
8) The velocity distribution based on the analytical model were compared with the measurements in situ carried out in naturally grouped vegetation in the floodplain of the Warta river, and laboratory rectilinear channel at the Department of Hydraulic Engineering and environmental restoration at the Warsaw University of Life Sciences. Considerable uniformity measurement with calculated results of in situ measurements shows that the analytical model may be applied to define velocity distribution and hydraulic roughness with the flow through naturally grouped vegetation with different degrees of submergence.

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Model analityczny rozkładu prędkości i szorstkości hydraulicznej przy przepływach w korytach i dolinach rzecznych porośniętych roślinnością

Streszczenie

W pracy przedstawiono, wyznaczone z bilansu sił, nowe rozwiązanie analityczne pionowego rozkładu prędkości i szorstkości hydraulicznej z roślinnością zatopioną i nie zatopioną. Dla tego rodzaju roślinności opracowano różne modele turbulencji. Wymienione rozwiązanie analityczne opisuje przepływ wody przez roślinność o różnej strukturze gęstości jej elementów w kilku warstwach. Przeprowadzone obliczenia prędkości wykazują bardzo dobrą zgodność z wynikami pomiarów w warunkach koryta naturalnego rzeki Warty oraz z wynikami pomiarów w korytach hydraulicznych. Stwarza to potencjalną możliwość szerokiego zastosowania prezentowanego rozwiązania w praktyce inżynierskiej.